Surfing

NSW PHYSICS 5&6

Module 5 Advanced Mechanics
Module 6 Electromagnetism

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Module 5 Advanced Mechanics

Projectile Motion

CONTENT FOCUS Analyse the motion of projectiles by resolving the motion into horizontal and vertical components, making the following assumptions: a constant vertical acceleration due to gravity and zero air resistance.

CONTENT FOCUS Apply the modelling of projectile motion to quantitatively derive the relationships between the following variables: initial velocity, launch angle, maximum height, time of flight, final velocity, launch height and horizontal range. Solve problems, create models and make quantitative predictions by applying the equations of motion relationships for uniformly accelerated and constant rectilinear motion.

CONTENT FOCUS Conduct a practical investigation to collect primary data in order to validate the relationships derived for projectile motion.

Motion In Gravitational Fields

CONTENT FOCUS Apply qualitatively and quantitatively Newton's law of universal gravitation to determine the force of gravity between two objects: $F = \frac{GMm}{r^2}$.

CONTENT FOCUS Apply qualitatively and quantitatively Newton's Law of Universal Gravitation to investigate the factors that affect the gravitational field strength: $g = \frac{GM}{r^2}$.

CONTENT FOCUS Apply qualitatively and quantitatively Newton's Law of Universal Gravitation to predict the gravitational field strength at any point in a gravitational field, including at the surface of a planet.

CONTENT FOCUS Investigate the orbital motion of planets and artificial satellites when applying the relationships between the following quantities: gravitational force, centripetal force, centripetal acceleration, mass, orbital radius, orbital velocity and orbital period.

CONTENT FOCUS Predict quantitatively the orbital properties of planets and satellites in a variety of situations, including near the Earth and geostationary orbits, and relate these to their uses.

CONTENT FOCUS Derive quantitatively and apply the concepts of gravitational force and gravitational potential energy in radial gravitational fields to the concept of escape velocity: $v_{esc} = \sqrt{\frac{GM}{r}}$.

CONTENT FOCUS Derive quantitatively and apply the concepts of gravitational force and gravitational potential energy in radial gravitational fields to total potential energy of a planet or satellite in its orbit: $U = \frac{GMm}{r}$.

CONTENT FOCUS Derive quantitatively and apply the concepts of gravitational force and gravitational potential energy in radial gravitational fields to total energy of a planet or satellite in its orbit: $U = -\frac{GMm}{r}$.

Derive quantitatively and apply the concepts of gravitational force and gravitational potential energy in radial gravitational fields to energy changes that occur when satellites move between orbits.

Investigate the relationship of Kepler's Laws of Planetary Motion to the forces acting on, and the total energy of, planets in circular and non-circular orbits using: $v_o = \frac{2\pi r}{T}$ and $\frac{r^3}{r^2} = \frac{GM}{4\pi^2}$.

Circular Motion

Conduct investigations to explain and evaluate, for objects executing uniform circular motion, the relationships that exist between: centripetal force, mass, speed and radius of turn.

Analyse the forces acting on an object executing uniform circular motion for cars moving around horizontal circular bends
Analyse the forces acting on an object executing uniform circular motion for a mass on a string.

Analyse the forces acting on an object executing uniform circular motion for objects on banked tracks.

Solve problems, model and make quantitative predictions about objects executing uniform circular motion in a variety of situations, using the relationships: $a = \frac{|v|^2}{r}$, $\Sigma F = \frac{m|v|^2}{r}$ and $\omega = \frac{\Delta\theta}{t}$.

Investigate the relationship between the total energy and work done on an object executing uniform circular motion.

Investigate the relationship between the rotation of mechanical systems and the applied torque: $\tau = rF_\perp = |r|\,|F| \sin\theta$.

Module 6 Eletromagnetism

Charged Particles, Conductors and Electric and Magnetic Fields

Investigate and quantitatively derive and analyse the interaction between charged particles and uniform electric fields, including the electric field between parallel charged plates : $E = \frac{V}{d}$.

Investigate and quantitatively derive and analyse the interaction between charged particles and uniform electric fields, including acceleration of charged particles by the electric field: $F = ma$, $F = qE$.

Investigate and quantitatively derive and analyse the interaction between charged particles and uniform electric fields, including the work done on the charge: $W = qV$, $W = qEd$, $KE = \frac{1}{2}mv^2$.

Model qualitatively and quantitatively the trajectories of charged particles in electric fields and compare them with the trajectories of projectiles in a gravitational field.

Analyse the interaction between charged particles and uniform magnetic fields, including the acceleration, perpendicular to the field, of charged particles.
Analyse the interaction between charged particles and uniform magnetic fields, including the force on the charge: $F = qvB \sin\theta$.

CONTENT FOCUS: Compare the interaction of charged particles moving in magnetic fields to the interaction of charged particles with electric fields.

The Motor Effect

CONTENT FOCUS: Compare the interaction of charged particles moving in magnetic fields to other examples of uniform circular motion.

CONTENT FOCUS: Investigate qualitatively and quantitatively the interaction between a current carrying conductor and a uniform magnetic field ($F = BIL \sin\theta$) to establish conditions under which the maximum force is produced, the relationship between the directions of the force, magnetic field strength and current, and conditions under which no force is produced on the conductor.

CONTENT FOCUS: Conduct a quantitative investigation to demonstrate the interaction between two parallel current carrying wires.

CONTENT FOCUS: Analyse the interaction between two parallel current carrying wires ($\frac{F}{L} = \frac{\mu_0}{2\pi} \times \frac{I_1 I_2}{r}$) and determine the relationship between the International System of Units (SI) definition of an ampere and Newton's third law of motion.

Electromagnetic Induction

CONTENT FOCUS: Describe how magnetic flux can change, with reference to the relationship $\Phi = BA$.

CONTENT FOCUS: Analyse qualitatively and quantitatively, with reference to energy transfers and transformations, examples of Faraday's law and Lenz's law ($\varepsilon = -\frac{\Delta\Phi}{\Delta t}$), including but not limited to the generation of an electromotive force (emf) and evidence for Lenz's law produced by the relative movement between a magnet, straight conductors, metal plates and solenoids or changes in current in one solenoid in the vicinity of another solenoid.

CONTENT FOCUS: Analyse quantitatively the operation of ideal transformers through the application of $\frac{V_P}{V_S} = \frac{N_P}{N_S}$ and $V_P I_S = N_P N_S$.

CONTENT FOCUS: Evaluate qualitatively the limitations of the ideal transformer model and the strategies used to improve transformer efficiency, including but not limited to incomplete flux linkage and resistive heat production and eddy currents.
Analyse applications of step-up and step-down transformers, including but not limited to the distribution of energy using high voltage transmission lines.

Applications of the Motor Effect

CONTENT FOCUS: Investigate the operation of a simple DC motor to analyse the functions of its components and the production of a torque ($\tau = nBA \cos\theta$) and the effects of back emf.

CONTENT FOCUS: Analyse the operation of simple DC and AC generators and AC induction motors.

CONTENT FOCUS: Relate Lenz's law to the law of conservation of energy and apply the law of conservation of energy to DC motors and magnetic braking.

Introduction

This book covers the Physics content specified in the NSW Physics Stage 6 Syllabus. Sample data has been included for suggested experiments to give you practice to reinforce practical work in class.

Each book in the *Surfing* series contains a summary, with occasional more detailed sections, of all the mandatory parts of the syllabus, along with questions and answers.

All types of questions – multiple choice, short response, structured response and free response – are provided. Questions are written in exam style so that you will become familiar with the concepts of the topic and answering questions in the required way.

Answers to all questions are included.

A topic test at the end of the book contains an extensive set of summary questions. These cover every aspect of the topic, and are useful for revision and exam practice.

Words To Watch

account, account for State reasons for, report on, give an account of, narrate a series of events or transactions.

analyse Interpret data to reach conclusions.

annotate Add brief notes to a diagram or graph.

apply Put to use in a particular situation.

assess Make a judgement about the value of something.

calculate Find a numerical answer.

clarify Make clear or plain.

classify Arrange into classes, groups or categories.

comment Give a judgement based on a given statement or result of a calculation.

compare Estimate, measure or note how things are similar or different.

construct Represent or develop in graphical form.

contrast Show how things are different or opposite.

create Originate or bring into existence.

deduce Reach a conclusion from given information.

define Give the precise meaning of a word, phrase or physical quantity.

demonstrate Show by example.

derive Manipulate a mathematical relationship(s) to give a new equation or relationship.

describe Give a detailed account.

design Produce a plan, simulation or model.

determine Find the only possible answer.

discuss Talk or write about a topic, taking into account different issues or ideas.

distinguish Give differences between two or more different items.

draw Represent by means of pencil lines.

estimate Find an approximate value for an unknown quantity.

evaluate Assess the implications and limitations.

examine Inquire into.

explain Make something clear or easy to understand.

extract Choose relevant and/or appropriate details.

extrapolate Infer from what is known.

hypothesise Suggest an explanation for a group of facts or phenomena.

identify Recognise and name.

interpret Draw meaning from.

investigate Plan, inquire into and draw conclusions about.

justify Support an argument or conclusion.

label Add labels to a diagram.

list Give a sequence of names or other brief answers.

measure Find a value for a quantity.

outline Give a brief account or summary.

plan Use strategies to develop a series of steps or processes.

predict Give an expected result.

propose Put forward a plan or suggestion for consideration or action.

recall Present remembered ideas, facts or experiences.

relate Tell or report about happenings, events or circumstances.

represent Use words, images or symbols to convey meaning.

select Choose in preference to another or others.

sequence Arrange in order.

show Give the steps in a calculation or derivation.

sketch Make a quick, rough drawing of something.

solve Work out the answer to a problem.

state Give a specific name, value or other brief answer.

suggest Put forward an idea for consideration.

summarise Give a brief statement of the main points.

synthesise Combine various elements to make a whole.

SURFING

NSW PHYSICS 5&6

Module 5

Advanced Mechanics

Content Focus

In this module you will:

- Describe and analyse qualitatively and quantitatively circular motion and motion in a gravitational field, in particular, the projectile motion of particles.
- Explain and analyse motion in one dimension at constant velocity or constant acceleration.
- Extend your study of motion into examples involving two or three dimensions that cause the net force to vary in size or direction.
- Develop an understanding that all forms of complex motion can be explained by analysing the forces acting on a system, including the energy transformations taking place within and around the system.
- Apply new mathematical techniques to model and predict the motion of objects within systems. You will examine two-dimensional motion, including projectile motion and uniform circular motion, along with the orbital motion of planets and satellites, which are modelled as an approximation to uniform circular motion.
- Engage with all the Working Scientifically skills for practical investigations involving the focus content to examine trends in data and to solve problems related to advances mechanics.

CONTENT FOCUS

Analyse the motion of projectiles by resolving the motion into horizontal and vertical components, making the following assumptions: a constant vertical acceleration due to gravity and zero air resistance.

1 Projectile Motion

Galileo and projectiles

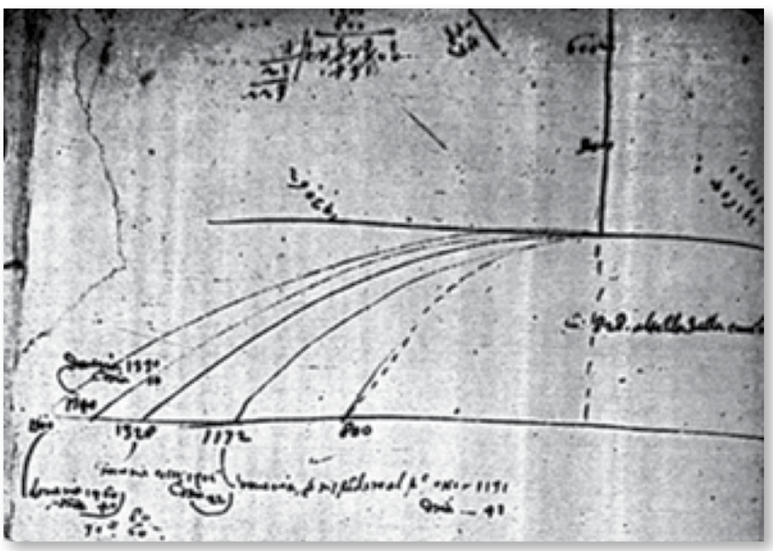
Galileo was one of the first scientists to study moving objects. He reputedly dropped masses from the Leaning Tower of Pisa to show that all objects fall at the same rate.

He also studied the relationship between the vertical and horizontal components of the velocity of a projectile by dropping a cannonball from the mast of a moving ship. The ball landed at the base of the mast and was not 'left behind' as those who supported Aristotle's ideas of motion believed.

In this way he showed that the components of the motion of a projectile were independent of each other.

In essence, Galileo's work showed that:

1. The horizontal motion of a moving object is not subject to gravitational forces, and therefore experiences no acceleration.
2. The vertical motion of an object is affected by the downwards force of gravity which gives it an acceleration of 9.8 m s^{-2} (slightly varying depending on where the object is).



Part of Galileo's original analysis of projectile motion.

Galileo's analysis of projectiles

- Horizontal and vertical components of projectile motion are independent of each other.
- Horizontal motion of a moving object is not subject to gravitational forces, and therefore experiences no acceleration.
- Vertical motion of an object near the surface of the Earth is affected by the downwards force of gravity which gives it an acceleration of 9.8 m s^{-2}.

Projectile motion and Newton's equations of motion

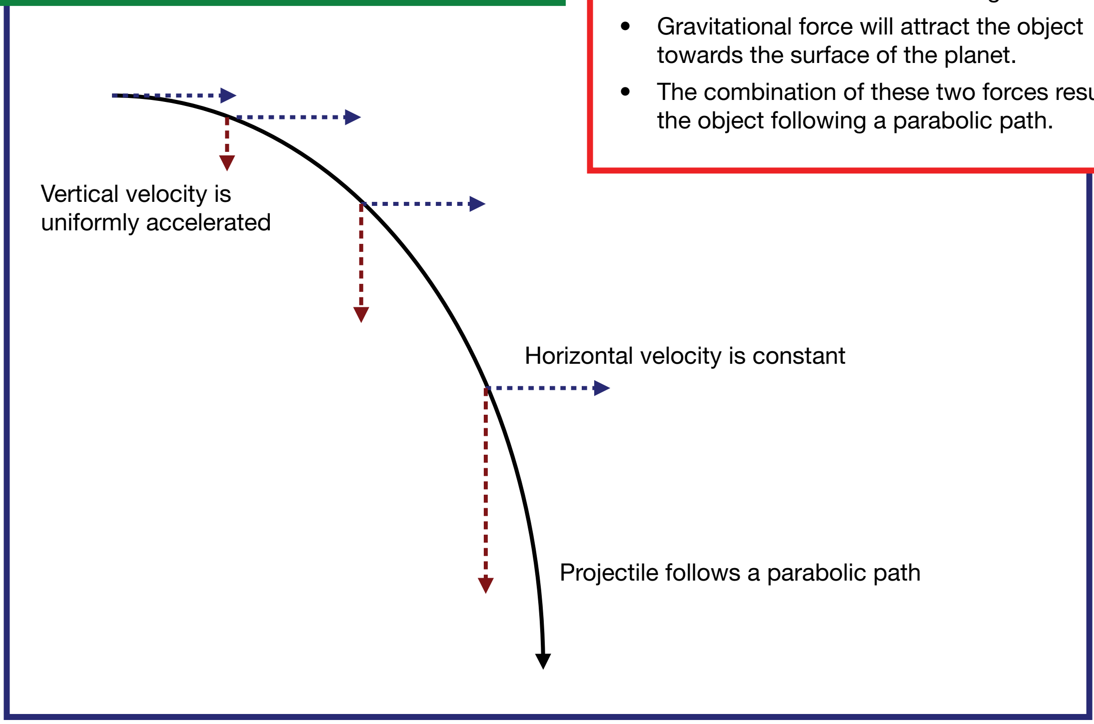
Three different types of projectile situations are detailed in the chapters that follow. Each type has specific data that assists in solving problems centred on that type. For all types, the basic equations used are derived from Newton's equations of linear motion. When applied to projectile motion, each equation is adapted to suit the characteristics of the horizontal and vertical components of the motion. The table summarises these equations.

Equations used in straight line motion	Horizontal component of motion	Vertical component of motion
$v = u + at$	$u_x = u \cos \theta$	$u_y = u \sin \theta$
$v^2 = u^2 + 2as$	$v_x = u_x$ ($a_x = 0$)	$v_y = u_y + a_y t$
$s = ut + \frac{1}{2}at^2$	$v_x^2 = u_x^2$	$v_y^2 = u_y^2 + 2a_y \Delta y$
	$\Delta x = u_x t$	$\Delta y = u_y t + \frac{1}{2}a_y t^2$

A projectile is any object thrown or shot into the air at any angle.

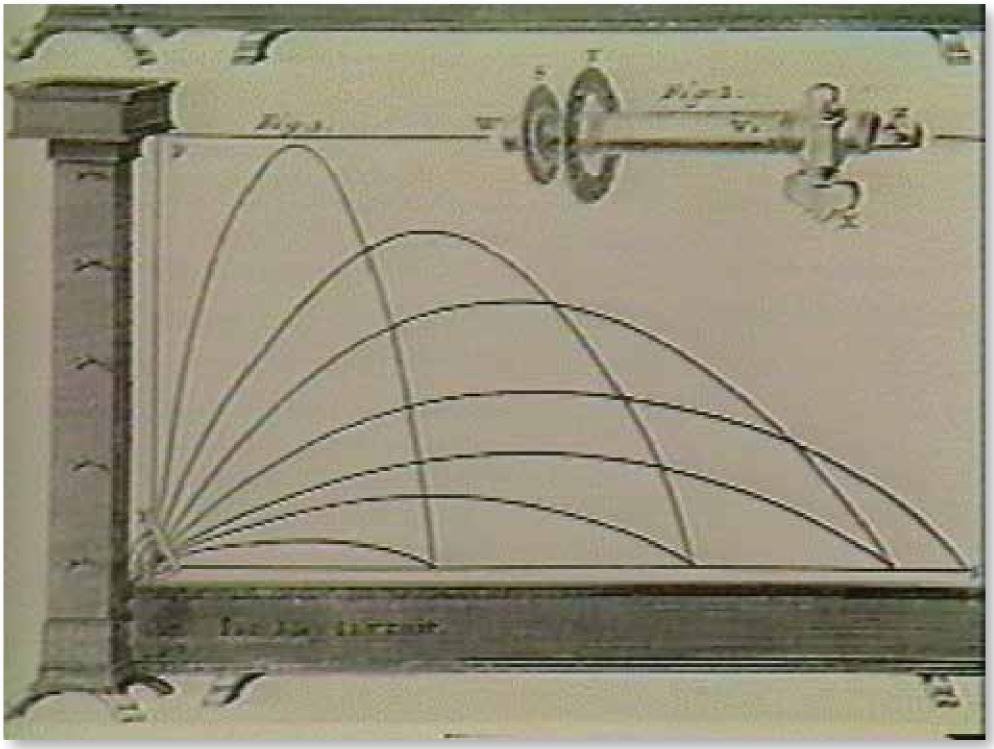
Trajectory of a projectile

- A projectile within a gravitational field is subject to its own inertia and to gravitational force.
- Inertia will cause it to continue to move in the direction of its motion in a straight line.
- Gravitational force will attract the object towards the surface of the planet.
- The combination of these two forces results in the object following a parabolic path.



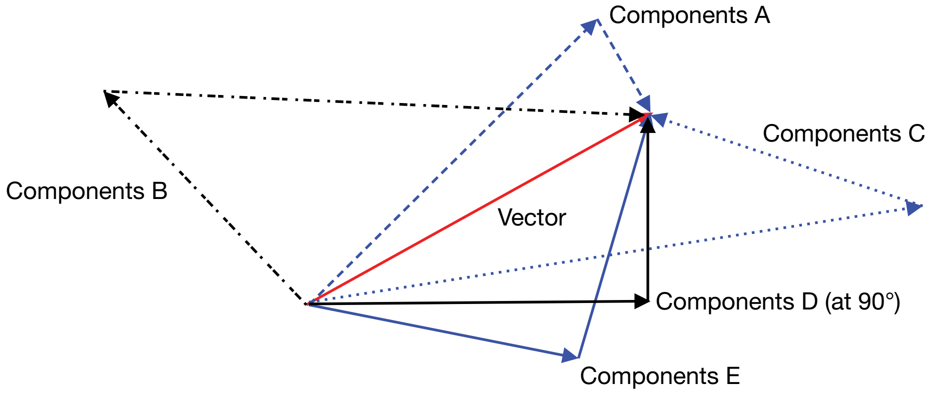
QUESTIONS

1. The diagram shows more of Galileo's analysis of projectile motion. In this diagram he is predicting that projectiles fired at complementary angles with the same velocities will have the same range.
   By considering projectiles fired at 40 m s^{-1} at both 30° and 60° to the horizontal, show that Galileo's prediction is supported.



2 Resolution Of Vectors – Revision

The projectile motion we analyse in this course involves analysis of vectors and the addition of vector components. Any vector has many components. The components are the vectors we add together to get the required vector. For example, the vector shown below (in red) has many pairs of components (shown in various blues) and one pair at right angles (black).



Normally though, when we refer to the components of a vector we specifically refer to the two vectors at 90° to each other, one **horizontal** and the other **vertical**, which would need to be added together to give that vector. In the diagram, these would be components D, drawn in black. When we **resolve** a vector into its components, then we are finding these two vectors at right angles.

Mathematically:

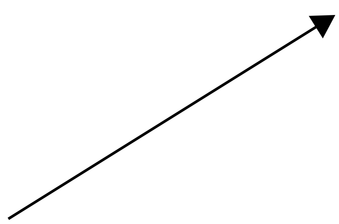
Horizontal component = vector cos θ

Vertical component = vector sin θ

QUESTIONS

1. Resolve the following vectors into their vertical and horizontal components. Use a scale where 1 cm = 5 N and state any directions as normal compass directions.

(a)



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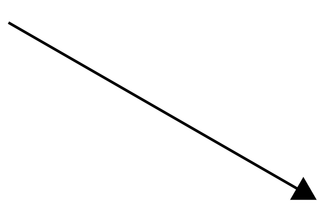
(b)



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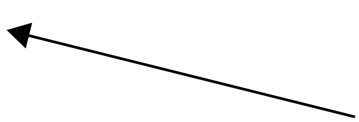
(c)



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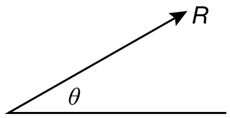
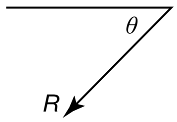
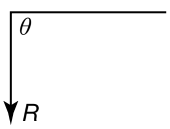
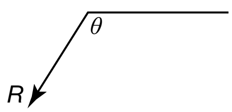
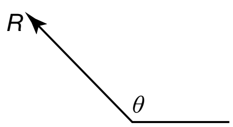
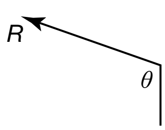
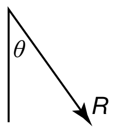
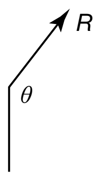
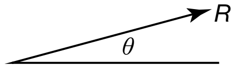
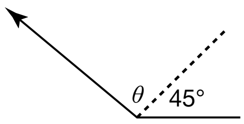
2. Resolve the following vectors into their vertical and horizontal components.

	Diagram	Angle	Resultant	Horizontal Component	Vertical Component	Answers
(a)		30°	6	A	B	A = B =
(b)		45°	C	10.6	D	C = D =
(c)		E	10	0	F	E = F =
(d)		120°	G	H	21.65	G = H =
(e)		I	J	12.95	12.5	I = J =
(f)		K	7.5	L	2.57	K = L =
(g)		36°25′	M	N	14.65	M = N =
(h)		O	72.05	42.65	P	O = P =
(i)		15°12′	Q	0.66	R	Q = R =
(j)		S	236	T	143.7	S = T =

Apply the modelling of projectile motion to quantitatively derive the relationships between the following variables: initial velocity, launch angle, maximum height, time of flight, final velocity, launch height and horizontal range.
Solve problems, create models and make quantitative predictions by applying the equations of motion relationships for uniformly accelerated and constant rectilinear motion.

3 Projectile Motion Problems 1

Object projected from a horizontal surface

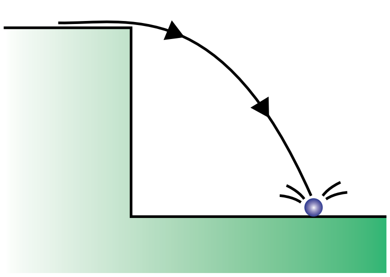
Special considerations:

- Initial vertical velocity = 0
- $\theta = 0°$

QUESTIONS

For each of the problems below, consider a projectile dropped vertically out of a moving object such as a plane, thrown horizontally out from the top of a cliff, or fired horizontally and landing on a surface as shown in the diagram. For each find any of the quantities which are not in the given data (not necessarily in the order given).

(a) The initial velocity of the projectile.
(b) Its initial horizontal velocity.
(c) Its initial vertical velocity.
(d) Its range.
(e) Its final horizontal velocity.
(f) Its final vertical velocity.
(g) Its final velocity.
(h) The time taken to fall.
(i) The height from which it was thrown or dropped.
(j) Its velocity 3 seconds after dropping.
(k) Its height 5 seconds after dropping.



1. A lifeboat is dropped from a plane moving at 140 m s^{-1} from a height of 1102.5 m.
2. A box of supplies is dropped from a helicopter moving at 80 m s^{-1}. They hit the ground 9.0 s later.
3. A ball is thrown horizontally from the top of a building and lands in a bucket on the ground 50 m in front of the building 3.0 s later.
4. A rock is thrown horizontally out from the top of a 147 m cliff. It hits the ground 80 m from the base of the cliff.
5. A ball thrown horizontally out from the top of a cliff hits the ground 6 s later at 30° to the vertical, and moving at 67.9 m s^{-1}.
6. A cannonball is fired horizontally from a castle. It hits its target 150 m away after 7.5 s.
7. An arrow is fired horizontally at the centre of a target 50 m away. Unfortunately, the archer made no allowance for gravity, and the arrow hit 0.8 m below centre.
8. A cannon fires a ball at 150 m s^{-1} which hits its target 675 m away.
9. A lifeboat is dropped from a plane moving at 80 km h^{-1}. It lands in the water 8 s later.
10. A projectile hits the centre of a target at an angle of 15° to the vertical 15 s after being dropped from a helicopter.

Now answer these questions

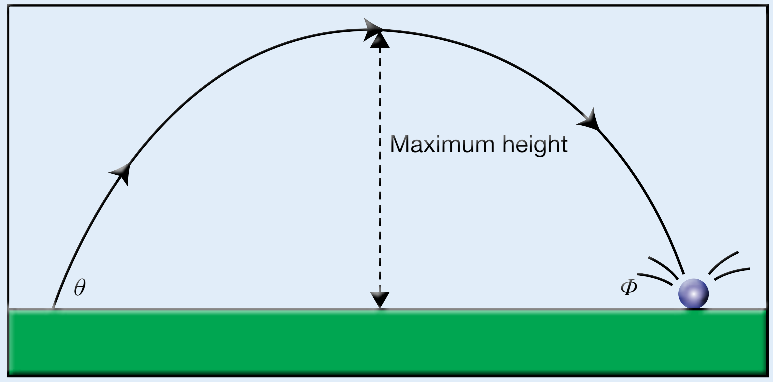
11. A car, moving at 30 m s^{-1}, goes over the edge of a cliff and into the water 58.8 m below.
    (a) Calculate the time it takes the car to hit the water.
    (b) Calculate the distance from the cliff that the car hits the water.
    (c) Calculate the speed of the car just as it hits the water.
12. A group of lemmings run over the edge of a 200 m cliff at 0.6 m s^{-1}.
    (a) Calculate their time to fall to the bottom of the cliff.
    (b) Calculate their velocity halfway down.
    (c) Calculate the time their speed will be 30.0 m s^{-1}.
    (d) Calculate the speed at which they hit the ground.

4 Projectile Motion Problems 2

Object thrown up and landing at same level

Special considerations:

- Total vertical displacement = 0.
- Vertical velocity at top of flight = 0.
- Time to rise = time to fall.
- Time to rise = half the time of flight.
- Speed at launch = speed at landing.
- Angle θ = angle Φ.
- Two halves of flight are symmetrical.
- Maximum height occurs when vertical velocity = 0.



QUESTIONS

For each of these problems, consider a projectile fired at an angle and landing on a surface at the same level as that from which it was fired, as shown in the diagram. For each problem find any of these quantities which are not in the given data (not necessarily in the order given).

(a) The initial velocity of the projectile.
(b) Its initial horizontal velocity.
(c) Its initial vertical velocity.
(d) Its range.
(e) Its maximum height.
(f) The time it takes to reach maximum height.
(g) Its time of flight.

1. Projectile fired at 20 m s^{-1} at 60° to the vertical.
2. Projectile is fired at 30 m s^{-1} and has a range of 79.5 m. It is in flight for 5.3 s.
3. Projectile is fired at 35.6 m s^{-1} at 55.8° to the horizontal and hits the ground 6 s later.
4. Projectile has a range of 318.2 m and rises to a maximum 60 m.
5. Projectile fired at 50° to the horizontal, rises 2.0 km.
6. Projectile's range is 2.5 km and rises for 30 s.
7. Projectile is fired at 200 m s^{-1} at 45°.
8. Projectile fired at 40° to the horizontal at 150 m s^{-1}.
9. Projectile rises to a height of 81.6 m after being fired at 60° to the horizontal.
10. Projectile hits a target 1.2 km away, 30 s after firing at 74.8°.
11. Projectile rises to a maximum height of 127.5 m and has a range of 408.2 m.
12. Projectile rises to 0.75 km, having been fired at 286.9 m s^{-1}.
13. Projectile fired at 400 m s^{-1} and rises for 20.4 s.
14. Projectile fired with minimum velocity to hit a target 3.0 km away 21 s after firing.
15. The vertical component of the velocity of a projectile is 147 m s^{-1} and the horizontal component of its velocity is 91.6 m s^{-1}.

Now answer these questions

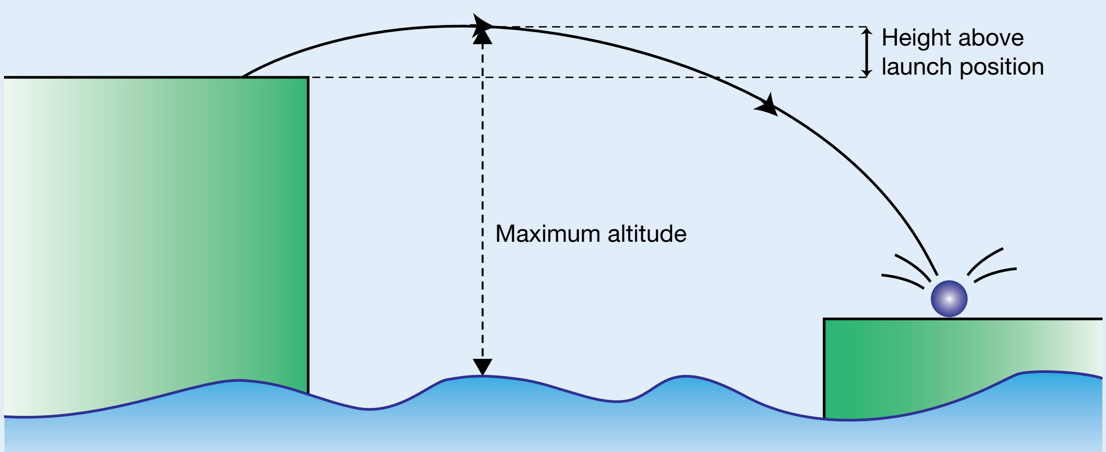
16. A projectile has a time of flight of 8.0 s and a range of 1120 m.
    (a) What maximum height does it reach?
    (b) At what velocity is it projected?
17. A projectile has a range of 750 m and reaches a maximum height of 58.8 m.
    (a) What is its time of flight?
    (b) What is its launch velocity?
18. A ball is hit at 30 m s^{-1} at 45° to the horizontal.
    (a) Calculate how far from the bat a fielder should stand to catch the ball at the same height as it was hit.
    (b) Calculate the time of flight if the fielder catches it.
    (c) Calculate how high the ball would rise after being hit.

5 Projectile Motion Problems 3

Object landing at a different level

Special considerations:

- Total vertical displacement = difference in height between the two levels.
- If target lower, then vertical displacement is negative (assume upwards direction positive).
- If target higher, vertical displacement positive.
- Vertical velocity at top of flight = 0.
- Time to rise *does not* equal time to fall.
- Time to rise *is not* half the time of flight.
- Speed at launch *does not* equal speed at landing.
- Angle of launch *does not* equal angle of landing.
- Two halves of flight are *not* symmetrical.
- Maximum height occurs when vertical velocity = 0.



QUESTIONS

For each of the problems below, consider a projectile fired at an angle and landing on a surface either below or higher than that from which it is fired, as shown in the diagram.

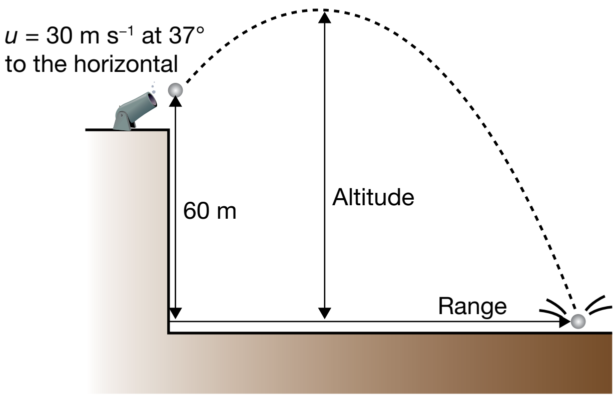
For each problem find any of these quantities which are not in the given data (not necessarily in the order given).

(a) The initial velocity of the projectile.
(b) Its initial horizontal velocity.
(c) Its initial vertical velocity.
(d) Its range.
(e) Its maximum height above the launch position.
(f) The time it takes to reach maximum height.
(g) Its time of flight.

1. A projectile is fired at 30° to the horizontal from the top of a cliff 200 m high. Its initial speed is 49 m s^{-1}.
2. A projectile is fired from the top of a 120 m high cliff at 25 m s^{-1}. It lands on the ground 6.4 s after firing.
3. A cannon is at the top of a 60 m high cliff firing at a castle on top of an adjacent cliff 110 m high. The cannon and castle are 200 m apart horizontally. The cannonballs hit 20 s after firing.
4. A cannonball is fired at 50 m s^{-1} from the top of a 200 m high cliff so that maximum range is achieved.
5. A boy throws a rock at 15 m s^{-1} from the top of a 75 m high cliff. The rock lands in the water at the bottom of the cliff 4.0 s later.
6. A projectile is thrown up from the top of a 60 m high cliff. It rises to a maximum height of 44.1 m above the cliff top. It hits the ground 76 m out from the base of the cliff.
7. A ball is thrown out from the edge of a 40 m high cliff with a velocity of 35.1 m s^{-1} at 30° to the horizontal.
8. A cannonball is fired at 40 m s^{-1} at 40° to the horizontal from the top of a 218.7 m cliff and hits a target 300 m from the base of the cliff.
9. A projectile fired up into the air from the top of a 75 m high cliff hits the ground 500 m out from the base 10 s later.
10. A cannon fires from the top of a 150 m high cliff at a castle 300 m from the base of the cliff. The ball hits the castle 15 s later.

Now answer these questions

11. A cannonball is fired at 40 m s^{-1} at an angle of 35° to the horizontal. Calculate the height at which the ball hits a vertical cliff 50 m away.
12. A player kicked a football from 30 m in front of the goalposts. The ball just cleared the crossbar which was 3.0 m above the ground 2.0 s later. Calculate:
    (a) The angle at which the ball was kicked.
    (b) The ball's initial speed.
13. A projectile is fired from the top of a cliff and reaches a height of 147 m above its launch point. It lands 980 m from the base of the cliff 20 s after launch. Calculate:
    (a) The height of the cliff.
    (b) The velocity at which the projectile was launched.
14. A rock is dropped into a well and the distance travelled is $5.0t^2$ m, where t is the time it took to fall. If the water splash is heard 2.75 seconds after the rock was dropped, and the speed of sound is 340 m s^{-1}, approximate the height of the well.
15. A mass of 5.0 kg is thrown vertically upwards from the top of a building at 22 m s^{-1}. It hits the ground 4.8 seconds later. What is the height of the building?
16. Consider the projectile shown in the diagram.



Find:
(a) Its time of flight.
(b) Its maximum height above the ground.
(c) Its range.

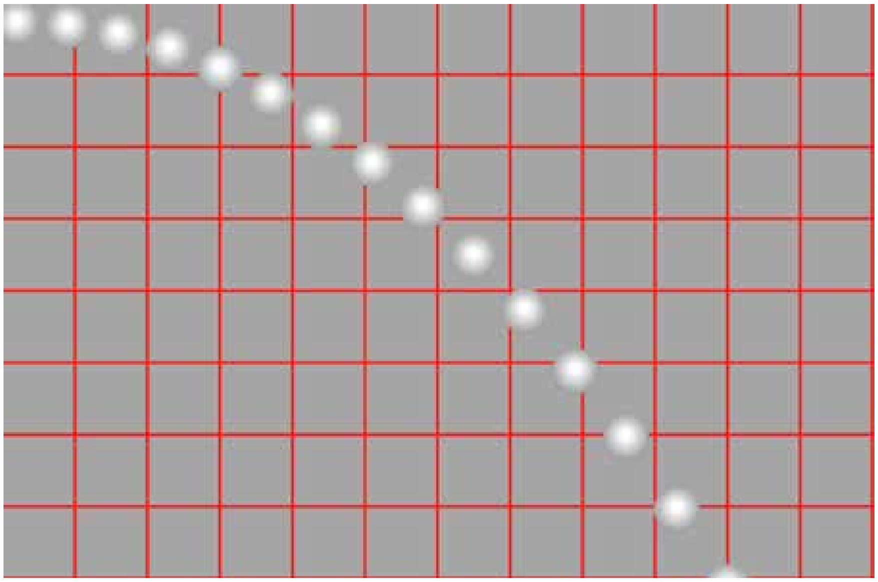
Conduct a practical investigation to collect primary data in order to validate the relationships derived for projectile motion.

6 Analysing Projectile Motion

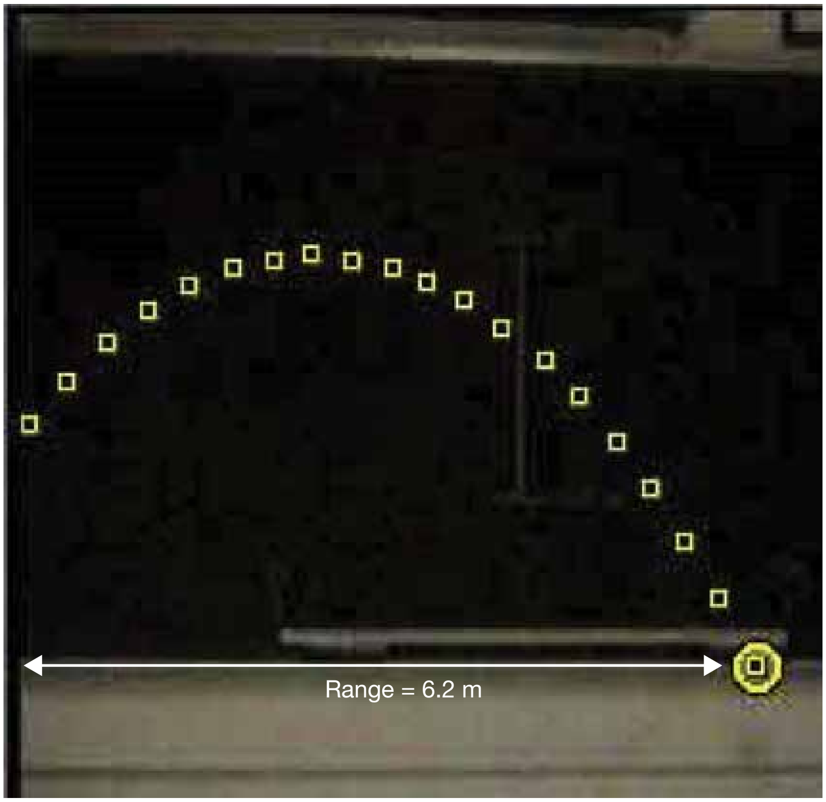
Analysing projectile motion diagrams

QUESTIONS

1. Analysis 1. The stroboscopic picture shows a projectile launched horizontally. If each grid square on the background sheet is 10 cm × 10 cm find:
   (a) The vertical displacement of the projectile.
   (b) Its time of flight.
   (c) The time interval between the flashes of the camera taking the picture.
   (d) The frequency of the stroboscope used.
   (e) The initial velocity of the projectile.



2. Analysis 2. The stroboscopic picture shows a projectile launched from a particular height above the floor in a high school gym, and landing on the floor. Find:
   (a) The angle at which the projectile was launched.
   (b) The vertical displacement of the projectile.
   (c) The maximum height above the launch position.
   (d) The initial vertical velocity of the projectile.
   (e) The time for the projectile to rise.
   (f) The time interval between the flashes of the camera.
   (g) The frequency of the stroboscope used to take the picture.
   (h) The horizontal velocity of the projectile.
   (i) The launch velocity of the projectile.

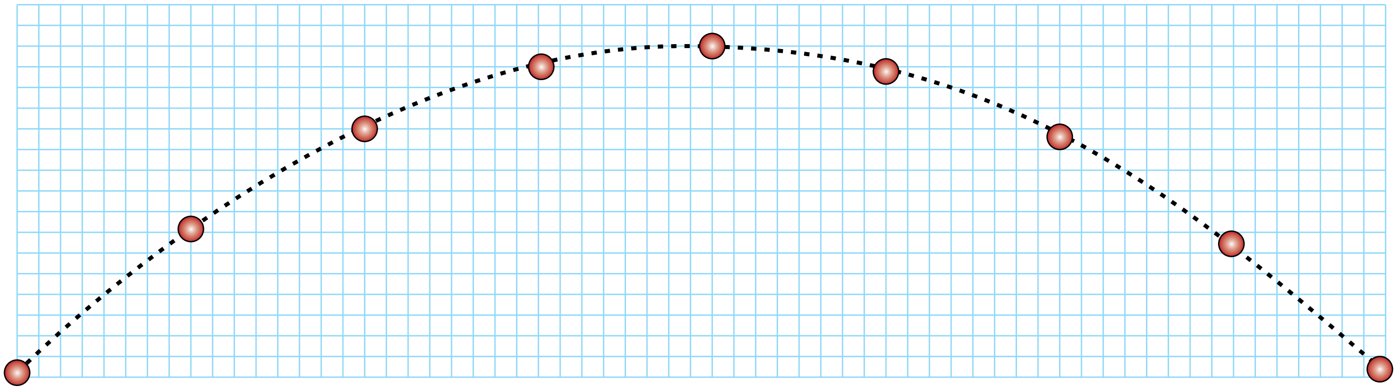


3. Analysis 3. The stroboscopic picture below shows an angry bird missing its target. The target tower is 30 m high from the ground to the top of the highest block.

Find the following.

(a) The angle at which the angry bird was launched.
(b) The vertical displacement of the angry bird above its launch position.
(c) The initial vertical velocity of the angry bird.
(d) The initial velocity of the angry bird.
(e) The time for the angry bird to rise.
(f) The time interval between the flashes of the camera.
(g) The frequency of the stroboscope used to take the picture.
(h) The horizontal distance between the angry bird and the tower.
(i) The launch velocity of the angry bird.

4. Analysis 4. The stroboscopic graph shows the flight of a projectile as plotted by a student.



If the graph grid is a 10 mm grid, find:

(a) The maximum height of the projectile.
(b) The range of the projectile.
(c) The vertical component of the projectile's velocity.
(d) The time of flight of the projectile.
(e) The time interval between plot points.
(f) The horizontal component of the projectile's velocity.
(g) The initial velocity of the projectile.

CONTENT FOCUS

Apply qualitatively and quantitatively Newton's law of universal gravitation to determine the force of gravity between two objects: $F = \frac{GMm}{r^2}$.

7 Mass and Weight

We often use the terms 'mass' and 'weight' – the latter often incorrectly. You do not, for example, 'weigh' 60 kg. Your 'mass' may certainly be 60 kg, but if that is the case, then your 'weight' is closer to 600 N! The difference lies in the following definitions.

Mass is a measure of the amount of matter in an object. Mass does not change, regardless of the position of the object in the Universe. Mass is measured in kilograms (kg) (or mg, g, tonnes etc).

The **weight** of an object is a measure of the force with which it is attracted by a gravitational force. The weight of an object changes depending on where it is in the Universe. Because weight is a force, it is measured in newtons (N).

Mass and weight are connected by the following equation.

$$W_F = mg$$

Weight = mass × acceleration due to gravity

Where W_F = weight in N

m = mass in kg

g = acceleration due to gravity

= 9.8 m s^{-2} (average value on Earth's surface at sea level)

QUESTIONS

1. Contrast mass and weight and give the units used to measure each.
2. Recall the equation connecting mass and weight.
3. Convert to weight: 100 grams, 2.5 kg, 20 kg, 1000 kg.
4. Convert to mass: 10 N, 100 N, 500 N, 10 000 N.
5. (a) Calculate the mass of an 85 kg person on the Moon. (Moon's gravity is about 1.6 m s^{-2}.)
   (b) Calculate this person's weight on the Moon.
   (c) Predict his weight on Earth.
   (d) Calculate the weight of a 55 kg astronaut on Earth.
   (e) Calculate the astronaut's mass and weight on the Moon.

6. The table shows the strength of gravity on heavenly objects compared to Earth.

Heavenly object	Gravitational field compared to Earth = 1
Earth	1.00
Mercury	0.41
Venus	0.91
Mars	0.38
Jupiter	2.53
Saturn	1.07
Uranus	0.88
Neptune	1.16
Pluto	0.064
Sun	28.1

   (a) Calculate the mass of a 1 kilogram block on each heavenly object.
   (b) Calculate its weight on each object.
   (c) Account for the mass and weight difference for each object.
   (d) List reasons for the difference in the force of gravity on each heavenly object.

Answers

1 Projectile Motion

1. For projectile at 30° to horizontal:
   $u_y = 40 \sin 30° = 20$ m s^{-1}
   From $v_{top} = 0 = u_y + gt = 34.641 - 9.8t$
   Time to rise = 2.0408 s
   So, time of flight = 4.0816 s
   Therefore range, $= u_x \times t_{flight} = 40 \cos 30° \times 4.0816 = 141.39$ m
   For projectile at 60° to horizontal:
   $u_y = 40 \sin 60° = 34.641$ m s^{-1}
   From $v_{top} = 0 = u_y + gt = 34.641 - 9.8t$
   Time to rise = 3.5347 s
   So, time of flight = 7.0796 s
   Therefore range, $= u_x \times t_{flight} = 40 \cos 30° \times 7.0696 = 141.39$ m
   Therefore, on the basis of this one calculation, Galileo's prediction is supported.

2 Resolution Of Vectors – Revision

1. (a) Horizontal = 21.6 N
   Vertical = 13.5 N
   (b) Horizontal = 20.35 N
   Vertical = 11.75 N
   (c) Horizontal = 27.9 N
   Vertical = 6.96 N
2. (a) A = 5.2
   B = 3.0
   (b) C = 15
   D = 10.6
   (c) E = 90
   F = 10
   (d) G = 25
   H = 12.5
   (e) I = 136
   J = 18
   (f) K = 110
   L = 7.05
   (g) M = 18.2
   N = 10.8
   (h) O = 144
   P = 58
   (i) Q = 0.68
   R = 0.18
   (j) S = 97.3
   T = 187.2

3 Projectile Motion Problems 1

All answers in order of information requested in the chapter (answers may differ slightly due to rounding errors).

1. 140 m s^{-1}, 140 m s^{-1}, 0, 2100 m, 140 m s^{-1}, 147 m s^{-1}, 203 m s^{-1} at 46.4° to horizontal, 15 s, 1102.5 m, 143.1 m s^{-1} at 11.9° to horizontal, 980 m
2. 80 m s^{-1}, 80 m s^{-1}, 0, 720 m, 80 m s^{-1}, 88.2 m s^{-1}, 119.1 m s^{-1} at 47.8° to horizontal, 9 s, 396.9 m, 85.2 m s^{-1} at 20.2° to horizontal, 274.4 m
3. 16.7 m s^{-1}, 16.7 m s^{-1}, 0, 50 m, 16.7 m s^{-1}, 29.4 m s^{-1}, 33.8 m s^{-1} at 60.4° to horizontal, 3 s, 44.1 m, 33.8 m s^{-1} at 60.4° to horizontal, 0 m
4. 14.6 m s^{-1}, 14.6 m s^{-1}, 0, 80 m, 14.6 m s^{-1}, 53.6 m s^{-1}, 55.6 m s^{-1} at 74.8° to horizontal, 5.5 s, 147 m, 32.8 m s^{-1} at 63.6° to horizontal, 24.5 m
5. 33.95 m s^{-1}, 33.95 m s^{-1}, 0, 203.7 m, 33.95 m s^{-1}, 58.8 m s^{-1}, 67.9 m s^{-1} at 60° to horizontal, 6.0 s, 176.4 m, 44.9 m s^{-1} at 40.9° to horizontal, 53.9 m
6. 20 m s^{-1}, 20 m s^{-1}, 0, 150 m, 20 m s^{-1}, 73.5 m s^{-1}, 76.2 m s^{-1} at 74.8° to horizontal, 7.5 s, 275.6 m, 35.6 m s^{-1} at 55.8° to horizontal, 153.1 m
7. 125 m s^{-1}, 125 m s^{-1}, 0, 50 m, 125 m s^{-1}, 3.92 m s^{-1}, 125.1 m s^{-1} at 1.8° to horizontal, 0.4 s, 0.8 m, 0, 0 m
8. 150 m s^{-1}, 150 m s^{-1}, 0, 675 m, 150 m s^{-1}, 44.1 m s^{-1}, 156.3 m s^{-1} at 16.4° to horizontal, 4.5 s, 99.2 m, 152.9 m s^{-1} at 11.1° to horizontal, 0 m
9. 22.2 m s^{-1}, 22.2 m s^{-1}, 0, 177.8 m, 22.2 m s^{-1}, 78.4 m s^{-1}, 81.5 m s^{-1} at 74.2° to horizontal, 8 s, 313.6 m, 36.8 m s^{-1} at 52.9° to horizontal, 191.1 m
10. 39.4 m s^{-1}, 39.4 m s^{-1}, 0, 590.8 m, 39.4 m s^{-1}, 147 m s^{-1}, 152.2 m s^{-1} at 75° to horizontal, 15 s, 1102.5 m, 49.2 m s^{-1} at 36.7° to horizontal, 980 m
11. (a) 3.46 s
    (b) 103.9 m
    (c) 45.3 m s^{-1}
12. (a) 6.4 s
    (b) 44.3 m s^{-1} at 89.2° down from horizontal
    (c) 3.06 s
    (d) 62.72 m s^{-1} at 89.5° (If you forgot to take the initial horizontal component into account your answer will be 62.610.)

4 Projectile Motion Problems 2

All answers in order of information requested in the chapter (answers may differ slightly due to rounding errors).

1. 20 m s^{-1} at 30° to horizontal, 17.3 m s^{-1}, 10 m s^{-1}, 35.3 m, 5.1 m, 1.02 s, 2.04 s
2. 30 m s^{-1} at 60° to horizontal, 15 m s^{-1}, 26 m s^{-1}, 79.5 m, 34.5 m, 2.65 s, 5.3 s
3. 35.6 m s^{-1} at 55.8° to horizontal, 20 m s^{-1}, 29.4 m s^{-1}, 120 m, 44.1 m, 3.0 s, 6.0 s
4. 57 m s^{-1} at 37° to horizontal, 45.4 m s^{-1}, 34.3 m s^{-1}, 318.2 m, 60.0 m, 3.5 s, 7.0 s
5. 258.5 m s^{-1} at 50° to horizontal, 166.1 m s^{-1}, 198 m s^{-1}, 6712 m, 2000 m, 20.2 s, 40.4 s
6. 296.9 m s^{-1} at 81.9° to horizontal, 41.7 m s^{-1}, 294 m s^{-1}, 2500 m, 4410 m, 30 s, 60 s
7. 200 m s^{-1} at 45° to horizontal, 141.4 m s^{-1}, 141.4 m s^{-1}, 4080 m, 1020.1 m, 14.43 s, 28.86 s
8. 150 m s^{-1}at 40° to horizontal, 115 m s^{-1}, 96.4 m s^{-1}, 2262 m, 474 m, 9.84 s, 19.67 s
9. 46.2 m s^{-1} at 60° to horizontal, 23.1 m s^{-1}, 40 m s^{-1}, 188.4 m, 81.6 m, 4.08 s, 8.16 s
10. 152.3 m s^{-1} at 74.8° to horizontal, 40 m s^{-1}, 147 m s^{-1}, 1200 m, 1102.5 m, 15 s, 30 s
11. 64 m s^{-1} at 50° to horizontal, 40 m s^{-1}, 50 m s^{-1}, 408.2 m, 127.5 m, 5.1 s, 10.20 s
12. 286.9 m s^{-1} at 25° to horizontal, 260 m s^{-1}, 121.2 m s^{-1}, 6433 m, 750 m, 12.37 s, 24.74 s
13. 400 m s^{-1} at 30° to horizontal, 346.4 m s^{-1}, 200 m s^{-1}, 14133 m, 2041 m, 20.4 s, 40.8 s
14. 176.1 m s^{-1} at 36° to horizontal, 143 m s^{-1}, 103 m s^{-1}, 3000 m, 543.3 m, 10.5 s, 21 s
15. 173.2 m s^{-1} at 58° to horizontal, 91.60 m s^{-1}, 147 m s^{-1}, 2748 m, 1102.5 m, 15 s, 30 s
16. (a) 78.4 m
    (b) 145.4 m s^{-1} at 15.5° to horizontal
17. (a) 6.93 s
    (b) 113.4 m s^{-1} at 17.4° to horizontal
18. (a) 91.8 m
    (b) 4.33 s
    (c) 22.9 m

5 Projectile Motion Problems 3

All answers in order of information requested in the chapter (answers may differ slightly due to rounding errors).

1. 49 m s^{-1} at 30° to horizontal, 42.4 m s^{-1}, 24.5 m s^{-1}, 396.9 m, 30.6 m, 2.5 s, 9.36 s